Objectives:

- Take derivatives of exponential functions.

Derivative of $f(x) = e^x$:

$$\frac{d}{dx}(e^x) = e^x$$

Example: Find $f'(x)$ for $f(x) = 5e^x$.

$$f'(x) = 5e^x$$

Example: Find $\frac{d}{dx}\big(f(x)\big)$ for $f(x) = 4x^7 + 2e^x$.

$$f'(x) = 28x^6 + 2e^x$$

Derivative of an exponential function with arbitrary base b:

$$\frac{d}{dx}(b^x) = \ln(b) \cdot b^x$$

Example: Find $f'(x)$ for $f(x) = 2^x$.

$$f'(x) = \ln(2) \cdot 2^x$$

Example: Find $\frac{d}{dx}\big(g(x)\big)$ for $g(x) = 4 \cdot 10^x + x^3 + e^x$.

$$g'(x) = 4 \cdot \ln(10) \cdot 10^x + 3x^2 + e^x$$

Example: Find the derivative of $s(t) = \pi^x - 3e^x + x^\pi + \pi^2$.

$$s'(t) = \ln(\pi) \cdot \pi^x - 3e^x + \pi x^{\pi-1} + 0 = \ln(\pi) \cdot \pi^x - 3e^x + \pi x^{\pi-1}$$

Question: What about $f(x) = e^{x^2}$?

Answer: We don't know how to handle this function yet...yikes!

Explanation of why if $f(x) = e^x$, then $f'(x) = e^x$:

First,

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{e^h - 1}{h}.$$

We can estimate this limit numerically:

$$\frac{e^{0.0001} - 1}{0.0001} \approx 1.00005 \qquad \frac{e^{-0.01} - 1}{-0.01} \approx 0.995$$

Let's guess that $f'(0) = 1$. Now,

$$f'(x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = e^x \lim_{h\to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x$$

Example: During the 2000's, the population of Hungary was modeled by

$$P(t) = 10.186(0.997)^t$$

($P(t)$ in millions of people, t in years since 2000). Assuming this model remains accurate:

1. What does the model say the population of Hungary was in the year 2000?

$$P(0) = 10.186(0.997)^0 = 10.186 \text{ (million people)}$$

2. What does the model predict for the population of Hungary in the year 2020?
   For the year 2020, we use $t = 20$:

$$P(20) = 10.186(0.997)^{20} = 9.592 \text{ (million people)}$$

3. How fast does the model predict the population will increase/decrease in 2020? (Include units)

$$P'(t) = 10.186(0.997)^t \cdot \ln(0.997)$$

$$P'(20) = 10.186(0.997)^{20} \cdot \ln(0.997) = -0.0288$$

   The population is *dropping* at a rate of about $29,000$ people per year

Example: Find the equation of the tangent line to $f(x) = 3e^x$ at $x = 1$.

To find the slope, we compute $f'(1)$:

$$f'(x) = 3e^x \qquad f'(1) = 3e$$

We know that the line intersects $f(x)$ at $x = 1$ so $\big(1, f(1)\big)$ is a point on the line. $f(1) = 3e$ so $(1, 3e)$ is a point on the line. We can use point-slope form to find that an equation for our tangent line is

$$y - 3e = 3e(x - 1)$$